Feasibility of Robot-Assistance Hand Physiotherapy in Post-Stroke Patients

Fatemeh Mohandesi (PhD Candidate)¹⁰, Alireza Mirbagheri (PhD)^{1,2}*¹⁰, Mohammad Mehdi Mirbagheri (PhD)^{1,3}, Noureddin Nakhostin Ansari (PhD)^{4,5}, Rouzbeh Kazemi (PhD)⁶

ABSTRACT

Background: Patients with experienced stroke have suffered from long-term disability, especially in their distal upper extremities. Physiotherapy programs are considered a proper treatment to overcome the complications caused by stroke. The use of robots in physiotherapy is also considered a newfound procedure as an alternative to conventional methods.

Objective: This study aimed to describe a feasibility test on a physiotherapy robot and evaluate the efficacy of the proposed device.

Material and Methods: In this experimental study, a 4-degrees-of-freedom robot was designed and fabricated for hand physiotherapy, which was tested on 17 and 4 post-strokes in the passive and active modes for the best efficiency. Additionally, the patient's hand spasticity was measured according to the Modified Ashworth Scale preand post-usage of the device.

Results: A total of 12 of 17 individuals could do the exercises and follow the instructions without any problem, and 8 of 12 individuals had a decrease in their spasticity. All 4 patients in active-assisted mode could fulfill the activity.

Conclusion: Physiotherapy based on a robot-assisted is considered a promising method with effective treatments for post-stroke patients, which can be a good alternative to routine methods of physiotherapy. However, more tests are needed to determine the rate of functions' restoration.

Keywords

Stroke; Rehabilitation; Hand; Robotically Physiotherapy; Modified Ashworth Scale

Introduction

troke is considered the third leading cause of death with long-term disability [1]. However, most of the stroke survivors have suffered from disability in their distal upper extremities [2], they do physiotherapy exercises, more than a third of individuals with the same stroke, who do not have strong and functional hands after a year. Moreover, no significant improvement is observed 6 to 8 months after the stroke, and limited treatment is known for hand weakness in patients with chronic stroke [3]. A follow-up study showed that only 6% of survivors were satisfied with their hands' functions 4 years after stroke [4].

Physiotherapy exercises are considered the most important approach to restoring hand functions [5] since the conventional methods of phys-

¹Department of Medical Physics and Biomedical Engineering, School of Medicine, Tehran University of Medical Sciences, Tehran, Iran

²Research Centre for Biomedical Technologies and Robotics (RCBTR), Advanced Medical Technologies and Equipment Institute (AMTEI), Tehran University of Medical Sciences, Tehran, Iran

³Department of Physical Medicine and Rehabilitation, Northwestern University, USA

⁴Research Center for War-Affected People, Tehran University of Medical Science, Tehran, Iran

⁵Department of Physiotherapy, School of Rehabilitation, Tehran University of Medical Science, Tehran, Iran

⁶PT and Technical Director of Tabassom Stroke Rehabilitation Center, Tehran, Iran

*Corresponding author: Alireza Mirbagheri Department of Medical Physics and Biomedical Engineering, School of Medicine, Tehran University of Medical Sciences, Tehran, Iran E-mail: a-mirbagheri@ tums.ac.ir

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iotherapy have some drawbacks, as follows: 1) the therapist needs to do many physical works, leading to fatigue and lack of carefulness in training the patients, 2) patients also become tired due to repetitive exercises, resulting in stopping treatment, and 3) a long time is needed for the patient's recovery, leading to increased treatment costs [6-8]. Different designed robots fabricated performing physiotherapy exercises on the hand, considered exoskeleton, such as soft exoskeleton robotic systems [9], Multi-Sensorial Immersive Dynamic Autonomous Systems (MIDAS) [10], and Hand Exoskeleton for Rehabilitation Objectives (HERO) [11]. In this field, other categories also work on devices with passive movements, such as a portable exoskeleton [12], a multi degree of freedom device for fingers [13], SPO (Script Passive Orthosis) [14], and an interactive rehabilitation robot [15], mostly fabricated for finger rehabilitation, including café [16], assist-on-fingers [17], ExoSkeleton for Index Finger [18], and IOTA (Isolated Orthosis for Thumb Actuation) [19].

Despite the variety of these robots, they are not completely used in clinical applications. This paper aimed to describe a feasibility test performed by a desktop-mounted device, designed, and fabricated based on clinical observations, leading to the training movements on the wrist and fingers. A feasibility test was performed in the two passive and active modes to

show the effect on clinical usage for the poststroke patients in the third stage of Brunnstrom evaluation, i.e., spasticity is the most severe.

Material and Methods

In this experimental study, the feasibility test was conducted by the proposed desktopmounted device, provided with the main four mechano-therapy movements, including passive, active-assisted, active, and active-resisted. The proposed desktop-mounted device is introduced with a simple mechanical structure and used for the wrist and four fingers for both hands with 4 degrees of freedom by using just one actuator. The device is designed to cover deformed hands with different degrees of spasticity and is also capable of changing the size and thickness of the fingers. Therefore, the test was done in the passive mode. For the active modes, the tests were performed on healthy individuals and patients with a lower degree of spasticity.

Physiotherapy Observations

Clinical needs were first assessed to identify the stroke impairments. Physiotherapy exercises on the hand and wrist were carefully observed to assess the needs of both therapists and patients. Also, the therapeutic movements were selected based on the therapist and the patient's needs. Figure 1 shows different types of task-oriented physical-therapy movements,



Figure 1: Different task-oriented physiotherapy movements

mostly utilized to open the wrist, and fingers were exerted on the hand. The therapist even had to stop his therapeutic activity and perform the operation of opening and stretching on the forearm and hand. Therefore, the spasticity of the flexor muscles was the main cause of disability in affected hands due to keeping the wrist and fingers flexed. Therefore, the desired device should be capable of exerting flexion-extension-tension on the wrist and fingers due to the therapists' and patients' needs.

Device Specifications

The functional requirements resulted from physiotherapy observations, which led to the construction of the device. Some sketches were then depicted in SolidWorks 2015 (Dassault-Systems, France, 1995). The final design was selected after examining the strengths and weaknesses of each design. Some criteria were considered to select the best design, as follows: 1) flexion-extension movements for wrist and fingers, 2) stretching movement on the flexor muscle, for spasticity reduction, 3) simple structure of the robot for easy usage, 4) use for both right, left, and adjustable hands for different finger sizes and thickness.

The desired model contains 4 DoFs, run by just one actuator. The four fingers are located in a cover, used for both the right and left

hand, and provided with a linear movement in the direction of bi-lateral linear guides and wagons, accompanied by a rotation about its longitude axis. Furthermore, the robot structure contains another rotational movement around the axis perpendicular to the palm. A Direct Current (DC) motor is provided to rotate the whole cover-linear guides set. The power transmission of the system is the linear guides and wagon, connected to the cover on both sides by two shafts. The forearm was also placed on a 30-degree stand and fastened by straps to avoid displacement. The driving unit is comprised of a DC motor (Maxon, 118755, Switzerland), a position controller (Maxon, Encoder MR type ML), a power supply (QUINT-PS/10-2866763), and an amplifier (Dynamic strain Amplifier, DN-AM100 DACELL, South Korea). In addition, the robot was equipped with a force sensor (Loadcell Zemic 1-S-B, Netherlands) to control the force exerted on the hand. A calibration was performed on the loadcell to ensure that the loadcell performs in a linear range. An encoder was attached to the DC motor to control the position of the robotic arm. For safety considerations, an emergency stop and an emergency button were provided, which could disable the device for any problems (Figure 2).

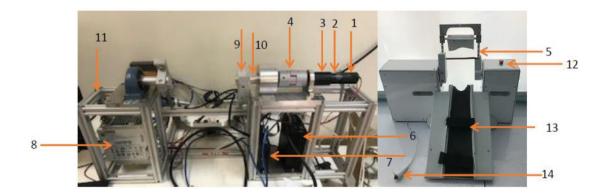


Figure 2: The robot platform 1: Encoder, 2: Direct-Current motor, 3: Reduction gearbox, 4: coupling mechanism, 5: Linear Guide, 6: Amplifier, 7: Position controller, 8: power supply, 9: load-cell, 10: Ball bearing, 11: mechanical structures, 12: Emergency stop, 13: Handset, 14: Emergency stop button

Control Procedure

The control system for the robot was programmed to produce all physiotherapy movements, including passive, active, active-assisted, and active resisted. The graphical user interface was created by C# in the visual studio program, (Microsoft Cooperation, Washington, United States). In the passive mode, the patient cannot move the robotic arm. Therefore, the device would exert the given movement on the wrist and hand. In activeassisted movement, the patient is capable of changing the robotic arm slightly, and the robot can detect this motion and help the patient to complete the motion. In the active mode, the patient moves the robotic arm without any contribution from the robot. Finally, the robot senses the patients' light motion and exerts a resistive force in the opposite direction of movement in the active-resisted mode, leading to overcoming the resistive force and completing the motion by the patients. The motion mode is selected based on the severity degree of the impairment.

Possible problems and issues should be identified in the robot since designing and fabrication of this robot aimed to have practical usage in treatment centers. Therefore, a questionnaire was designed to reflect the therapists' opinions during the use of the robot since the

therapists filled them out during performing the clinical tests. This questionnaire consisted of two sections, as follows: 1) questions about the ease of setting up the device in terms of hardware and software and 2) clinical questions about the proper physiological parameters, set on the graphical user interface, such as velocity, force, and tension time. The answers were classified into three levels, excellent, good, and weak. The therapists declared that the HandRoboHab was easily connected to a computer, and its software run without any problem. The adjustable clinical parameters were reasonable for stroke patients in any condition (Table 1).

Feasibility Test

Feasibility tests were conducted in the main modes: passive and active to show the usability of the proposed device. For this purpose, the loadcell calibration was first done.

The loadcell was placed in the force-gauge and imposed on the loadcell from the initial value of 0 in 5 N steps to a maximum value of 50 N to obtain the corresponding values (Figure 3).

The output voltage was recorded by EPOSE STUDIO software in terms of the torque imposed on the loadcell at each force exertion with the corresponding diagram. This proce-

Table 1: The questionnaire filled by therapists

Questions	Therapist1	Therapist2	Therapist3	Therapist4
Device appearance	Excellent	Excellent	Excellent	Excellent
The ease of connecting the device to the computer	Excellent	Excellent	Excellent	Excellent
The ease of running the software	Excellent	Excellent	Excellent	Excellent
Appropriate settings in graphical user interface	Excellent	Excellent	Excellent	Excellent
The ease of hand placement in the finger cover	Good	Good	Good	Good
The ease of fixing digit supporter	Good	Good	Good	Good
The efficiency of the device	Excellent	Good	Excellent	Good
Appropriate modes of movement	Excellent	Excellent	Excellent	Excellent
Therapist labor reduction	Excellent	Excellent	Excellent	Excellent
Suitable for replacement with the traditional methods	Excellent	Excellent	Excellent	Excellent



Figure 3: Loadcell calibration in force-gauge

dure was completed in the two directions of force imposing, and the diagrams were depicted. Equation 1 shows the relationship between the imposed torque and the output voltage in the real direction of force exertion to the loadcell, (Figure 4).

$$\tau = 0.0036 \text{V} - 0.9761 \tag{1}$$

The intended equation for the opposite direction of force imposing is written in (2), Figure 5.

$$\tau = -0.0036V + 0.7349 \tag{2}$$

In Equations (1) and (2), the τ is the imposed torque and V is the voltage corresponding to the torque.

The device was tested on 10 healthy individuals to understand possible software and hardware problems before testing the device on patients. These healthy individuals were practiced in all available modes with both

right and left hands, and no serious problems were observed.

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The device was also tested on the cases with the most severe amount of spasticity for better evaluation of the device's performance, safety, and clinical usability. Therefore, the patients, who were in the third stage of Brunnstrom, were selected for the passive mode test as they could not do the exercises actively. In this stage, the spasticity is maximum, and most of the patients refer to treatment centers. A recruitment criterion was prepared to deter-

mine the excluded subjects. The patients with flaccid hands and cognitive problems were excluded, and 17 individuals finally participated in the passive mode test, i.e., 10 and 7 individuals were left- and right-side affected cases, respectively. All patients did the exercises in 8 sessions, and individuals, who failed to work in the first session, could not continue

this process. The duration of training was dependent on the individual's strength. The test procedure was done after connecting the computer to the robot and running the software so that the patient sat on the back of the device with his elbow on the bottom strap, and the top strap was used to fix the wrist. An elastic cable was embedded to support the palm. The

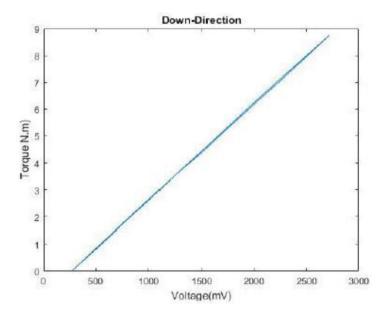


Figure 4: Diagrams of the calibrating loadcell in its real direction of force imposing

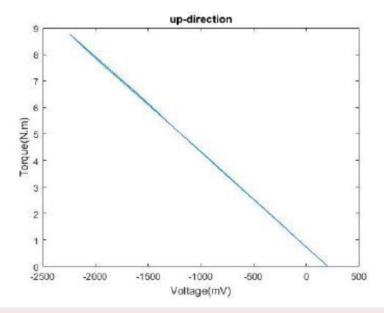


Figure 5: Diagrams of the calibrating loadcell in the opposite direction of force imposing

VI

fingers would be placed inside the cover and fixed according to the length and thickness of the fingers (Figure 6).

The Modified Ashworth scale (MAS) was measured before and after the training to identify the rate of therapeutic effect. Table 2 shows the specifications of the patients who participated in the passive mode, accompanied by the result of the test.

The patients with less spasticity were recruited for the active test. A total of 3 and 1 left- and right-side affected individuals were selected to train with Hand RoboHab in the active-assisted mode. The most important feature of the robot is the weight compensation of the robot arm in the active modes, showing the weight of the robot should not be imposed on the hand to have the easy-movement arm. Table 3 shows the case specifications in active-assisted mode.

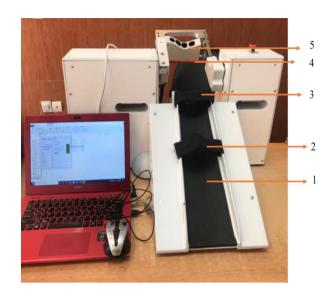


Figure 6: The setup of the test, 1) hand stand, 2) bottom strap, 3) top strap, 4) elastic cable, and 5) fingers cover

Table 2: The patients' specifications for passive test

Subjects	Affected-side	Sex	Age	Test result	Modified Ashworth Scale before training	Modified Ashworth Scale after training
1	Left	F	42	failed	4	-
2	Left	F	50	succeed	4	3
3	Left	F	70	succeed	4	4
4	Left	М	21	succeed	4	3
5	Left	М	54	succeed	4	3
6	Left	F	42	succeed	4	3
7	Left	М	40	succeed	4	3
8	Left	М	55	failed	4	-
9	Left	М	60	failed	4	-
10	Left	F	52	succeed	4	4
11	Right	F	40	succeed	4	3
12	Right	F	50	succeed	4	3
13	Right	М	56	succeed	4	4
14	Right	М	48	failed	4	-
15	Right	F	39	failed	4	-
16	Right	М	45	succeed	4	4
17	Right	F	57	succeed	4	3

Table 3: Patients' specifications for active-assisted test

Subjects	Affected- side	Sex	Age	Training set
1	Left	F	52	2 set
2	Left	F	43	3 set
3	Left	М	55	1set
4	Right	М	47	2 set

Results

A total of 7 of 10 left-side and 5 of 7 right-side affected patients could follow the instructions and be trained with the device without any problem. The other cases, which failed, faced physiological problems, such as wrist stiffness and pain in the shoulder. Also, 8 of 12 patients, who were successful in training with the robot, showed a decrease in Modified Ashworth Scale (MAS), i.e., their hand spasticity reduced to some extent, as shown in Table 1.

All these 4 patients of the active-assisted mode test could do the exercises without any problems and confirmed that they did not feel the robot arm weight. The number of training sets is dependent on the individual's hand strength. In this mode, the margin was adjustable as a special location in the workspace, in which the robot started to apply force to the hand.

Discussion

Brain traumas, such as stroke affect the body, including loss of hand function, and physical therapy is employed to restore hand function. Physiotherapy robots are designed and fabricated to have repetitive and regular training [20]. Despite the variety of these models, they have not been widely used in treatment centers due to ignoring the needs of both therapists and patients. In this paper, a feasibility test, performed by a robotic device HandRoboHab, is described to show its efficacy and usability

in physical therapy centers. The robot contains 4 degrees of freedom based on clinical observation, in which the optimal therapeutic movement is selected. This is a simply structured robot with the ability to perform four mechano-therapy movements to the fingers and wrist, passive, active, active-assisted, and active-resisted. The average length and thickness of fingers knuckles in [21] were used in software-design phase, which made the device usable for a wide range of patients. Thus, the robot is adjustable for different finger sizes. Unlike previous robots that could only be used for one finger, or one hand [16, 17], this robot can be used for both hands, due to the hand cover design. The structure of the robot is very simple, in which only one actuator was employed to run the four degrees of freedom. This feature is less common in similar robots since one actuator should be provided at least for every degree of freedom.

Based on the obtained results, most of the patients with severe spasticity could do the robot-assisted physiotherapy since the robotassisted training affects rehabilitation of the distal upper limbs, i.e., the proposed device is well-adjustable for different finger sizes. Furthermore, a relatively wide group of patients with different degrees of spasticity could participate in the test, showing that this device is proper for any neuro-muscular disorders with different complications. Further, all patients became more emotionally motivated to continue the treatment process. The robot had therapeutic effects on the patients, due to a relative reduction in spasticity rate. On the other hand, the therapists were surveyed about the ease of using the device, via a questionnaire, and they all claimed that manipulation of the robot was easy with software and hardware. Besides, the items, such as the velocity of the robotic arm and stretching time were appropriate for a vast range of patients, showing the device could eliminate the difficulties in conventional physiotherapies, such as the labor job of the therapists. The robot-assisted physiotherapy can produce repetitive and exact movements on the hands without any limitations.

Conclusion

The current study aimed to evaluate the efficacy of a physiotherapy robotic system on the recovery of motor deficiencies caused by brain trauma. The preliminary findings showed that employing the device is easy in terms of software and hardware. The most significant result indicated that this robotic physiotherapy system was capable of improving the hands' functions to some extent. Further experimental investigations are needed to show more improvements in the damaged hands based on Fugl-Meyer or ARAT assessment.

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Authors' Contribution

F. Mohandesi performed the tests and wrote the whole manuscript. A. Mirbagheri defined and managed the project. MM. Mirbagheri supervised the project. N. Nakhostin Ansari conducted the interpretations of the results. R. Kazemi managed the participants in the tests. All the authors read, modified, and approved the final version of the manuscript.

Ethical Approval

The Ethics Committee of Tehran University of Medical Sciences approved the protocol of the study (Ethic cod: IR.TUMS.REC.1394.1505).

Informed consent

To comply with established principles of research ethics, prior to the beginning, participants were provided with an informed consent form, elaborating on the research objectives and procedures. The form explains the participant's right to abandon, in any stage of the tests. Participants were also assured that their information will remain confidential and all participants signed the form.

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Conflict of Interest

None

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